EE 362: Signals and Systems Name: _____

Spring 2019

Homework #2 (Due: Tuesday, February 5, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

Refer to your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).

1. Work out problem 1.1 part (i) [2 points]

2. Work out problem 1.4 part (a), (b), and (c). [2 points]

3. Work out problem 1.7 part (a) to (g). [3 points]

4. Work out problem 1.17. [3 points]

5. Work out problem 1.19. [4 points]

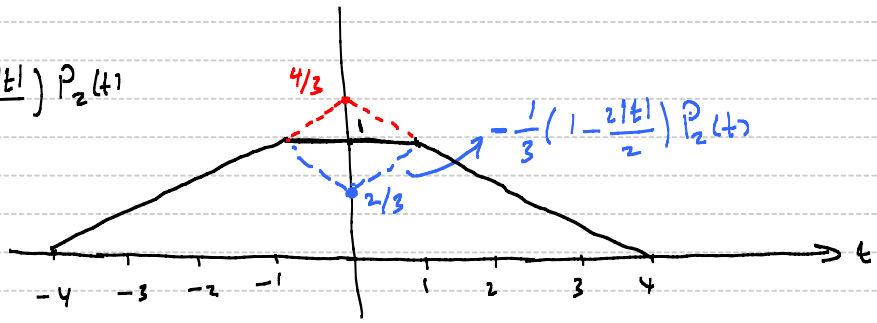
6. Work out problem 1.20. [4 points]

7. Work out problem 1.29. [4 points]

(1) 1.1 – (i)

a) $P_4(t) + P_2(t)$

b) $\frac{4}{3}\left(1-\frac{2|t|}{8}\right)P_8(t) - \frac{1}{3}\left(1-\frac{2|t|}{2}\right)P_2(t)$



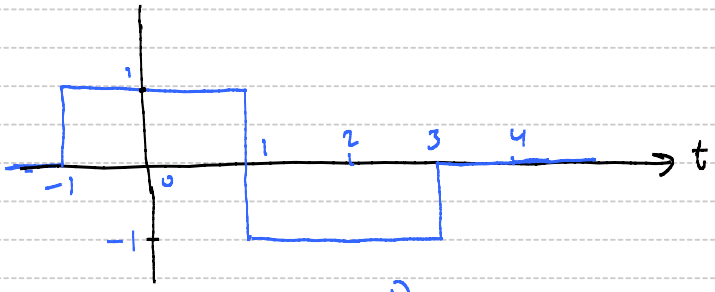
c) $2P_{12}(t) + 2P_6(t) + 2\left(1-\frac{2|t|}{6}\right)P_6(t)$

d) $4P_4(t) - 2\left(1-\frac{2|t|}{4}\right)P_4(t)$

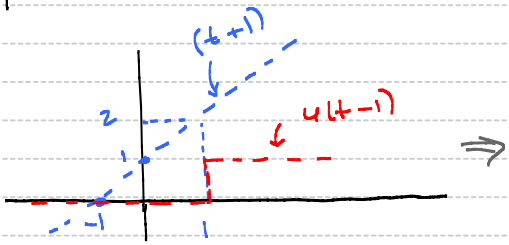
e) $P_1(t-\frac{1}{2}) + P_1(t-2.5) + P_1(t-4.5) + \cdots$

OR $\sum_{k=0}^{\infty} P_1(t-0.5-2k)$

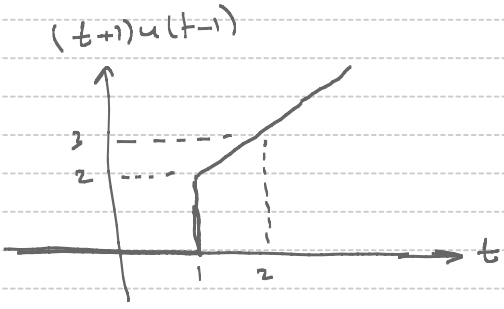
(2) a)



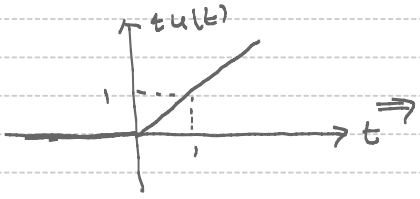
b) $(t+1)u(t-1)$ →



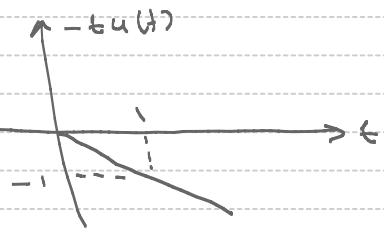
⇒



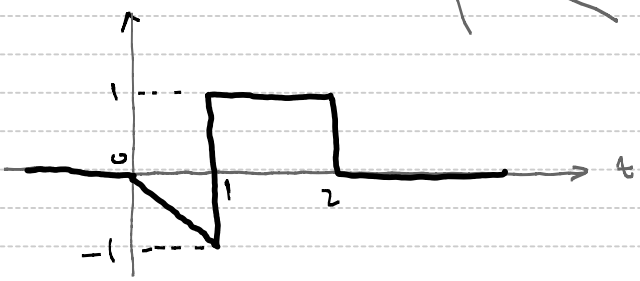
$tu(t)$ →



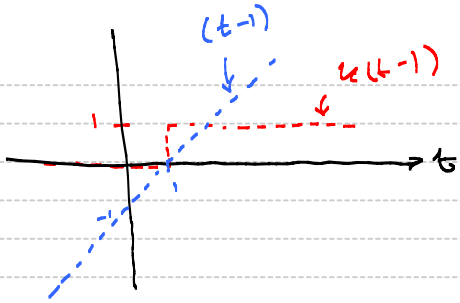
⇒ $-tu(t)$ ⇒



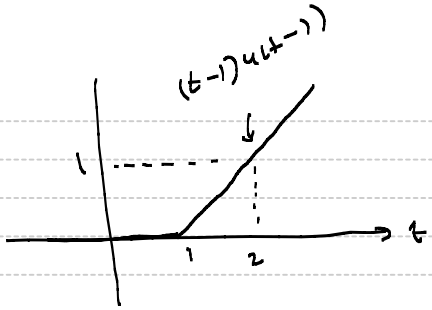
$(t+1)u(t-1) - tu(t) - u(t-2)$ ⇒



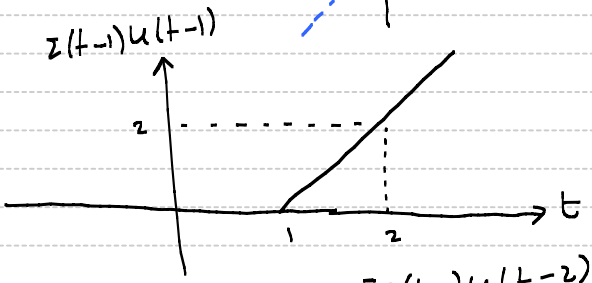
c) $2(t-1)u(t-1) \rightarrow$



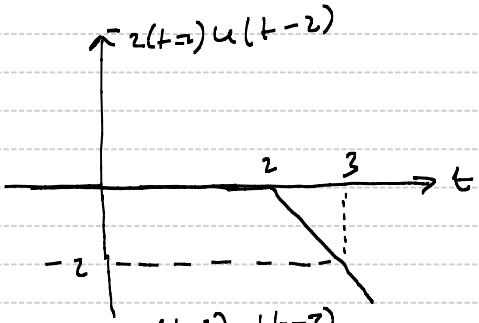
$\Rightarrow$



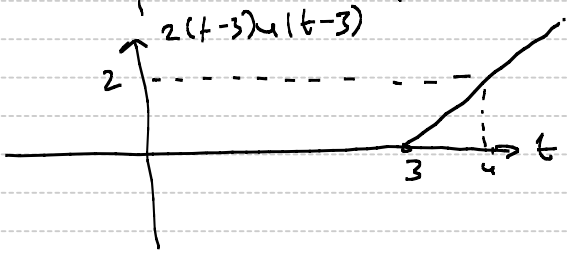
$\Rightarrow$



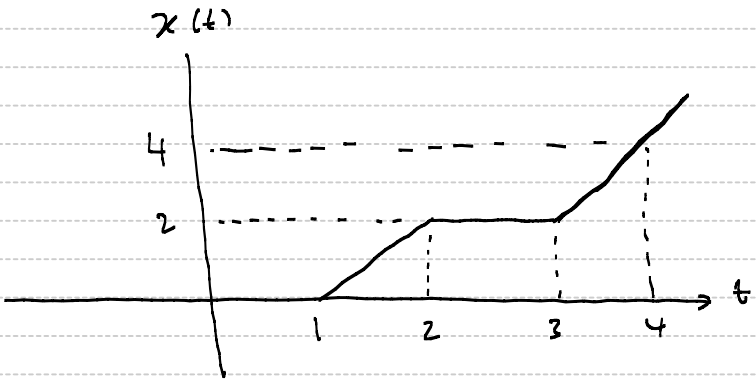
$-2(t-2)u(t-2) \Rightarrow$



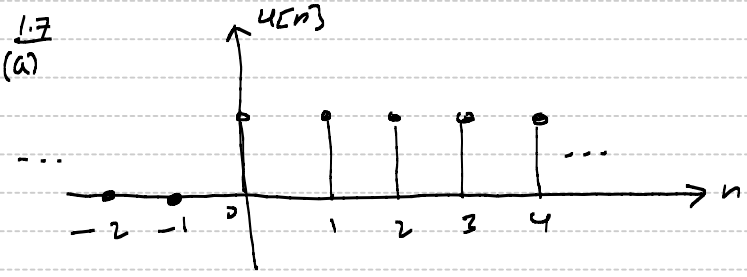
$2(t-3)u(t-3) \Rightarrow$



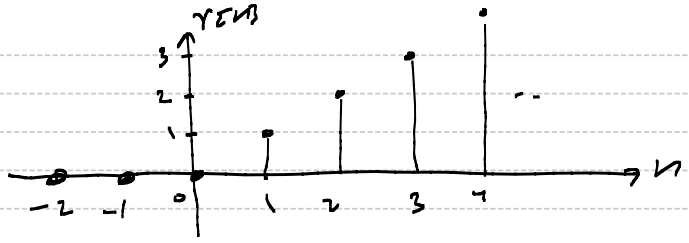
$\therefore$



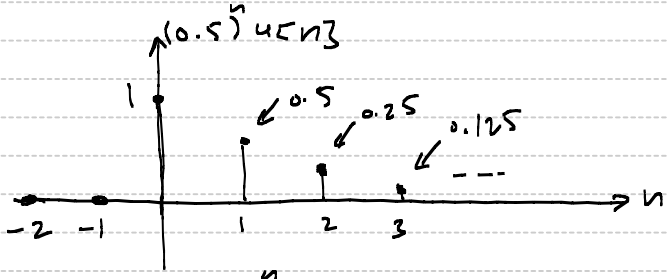
(3) 1.7 (a)



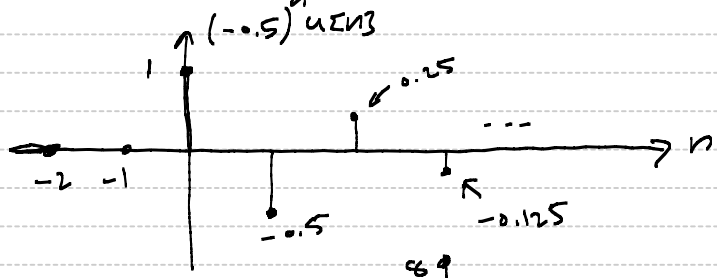
(b)


r[n]
3
2
1
-2
-1
0
1
2
3
4
n

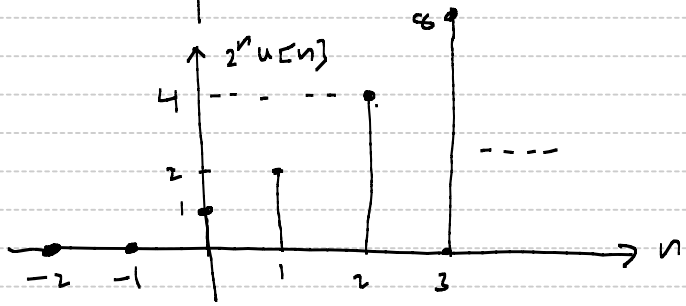
(c)


$(0.5)^n u[n]$
1
0.5
0.25
0.125
-2
-1
1
2
3
n

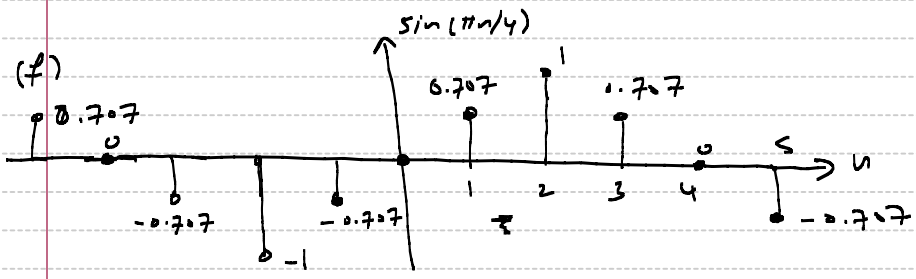
(d)


$(-0.5)^n u[n]$
1
0.25
-0.5
-0.125
-2
-1
n

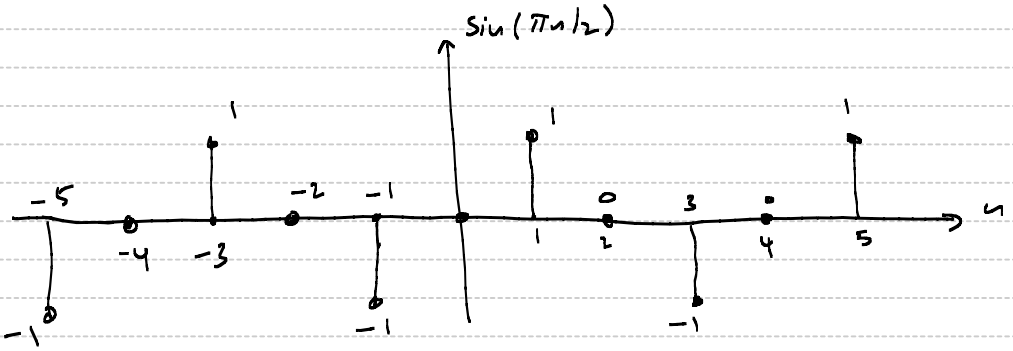
(e)


$2^n u[n]$
16
4
2
1
-2
-1
1
2
3
n

(f)


$\sin(\pi n/4)$
0.707
1
0.707
0.707
0
0
5
n
-0.707
-1
-0.707
-0.707
1
2
3
4

(g)


$\sin(\pi n/2)$
1
1
1
-5
-2
-1
0
3
n
-4
-3
1
2
4
5
-1
-1
-1

(4) 1.17

(a) Since the output depends only on the input at time "t", the system is caused and memoryless

(b) Same answer as that in part (a)

(c) Same answer as that in Part (a)

(d) Same answer as that in part (a)

(e) Caused since it does not depend on future input. It has a memory since the integral depends on the input from 0 to t

(f) Same answer as that in part (e)

(5) 1.19

(a) $x_1(t) \to y_1(t) = |x_1(t)|$
$x_2(t) \to y_2(t) = |x_2(t)|$
$\Rightarrow a x_1(t) + a x_2(t) \to y_3(t) = |a x_1(t) + b x_2(t)|$

$a y_1(t) + b y_2(t) = a|x_1(t)| + b|x_2(t)|$

Since $|a x_1(t) + b x_2(t)| \neq a|x_1(t)| + b|x_2(t)|$, the system is not linear

(b) $x_1(t) \to y_1(t) = e^{x_1(t)}$
$x_2(t) \to y_2(t) = e^{x_2(t)}$
$\Rightarrow a y_1(t) + b y_2(t) = a e^{x_1(t)} + b e^{x_2(t)}$

$a x_1(t) + b x_2(t) \to y_3(t) = e^{a x_1(t) + b x_2(t)}$

Since $e^{a x_1(t) + b x_2(t)} \neq a e^{x_1(t)} + e^{x_2(t)}$, the system is not linear

(c) $x_1(t) \to y_1(t) = (\sin t) x_1(t)$
$x_2(t) \to y_2(t) = (\sin t) x_2(t)$
$\Rightarrow a y_1(t) + b y_2(t) = a(\sin t) x_1(t) + b(\sin t) x_2(t)$

$a x_1(t) + b x_2(t) \to y_3 = (\sin t)(a x_1(t) + b x_2(t))$

Since $(\sin t)(a x_1(t) + b x_2(t)) = a(\sin t) x_1(t) + b(\sin t) x_2(t)$

the system is linear

(d)

$$x_1(t) \rightarrow y_1(t) = \begin{cases} x_1(t), & |x_1(t)| \le 10 \\ 10, & |x_1(t)| > 10 \end{cases}$$

$$x_2(t) \rightarrow y_2(t) = \begin{cases} x_2(t), & |x_2(t)| \le 10 \\ 10, & |x_2(t)| > 10 \end{cases}$$

$$ax_1(t) + bx_2(t) \rightarrow y_3(t) = \begin{cases} ax_1(t) + bx_2(t), & |ax_1(t) + bx_2(t)| \le 10 \\ 10, & |ax_1(t) + bx_2(t)| > 10 \end{cases}$$

$$ay_1(t) + by_2(t) = \begin{cases} ax_1(t), & |x_1(t)| \le 10 \\ 10, & |x_1(t)| > 10 \end{cases} + \begin{cases} bx_2(t), & |bx_2(t)| \le 10 \\ 10, & |bx_2(t)| > 10 \end{cases}$$

∴ The system is <u>not linear</u>

(e)

$$x_1(t) \rightarrow y_1(t) = \int_0^t (t-\lambda)\, x_1(\lambda)\, d\lambda$$

$$x_2(t) \rightarrow y_2(t) = \int_0^t (t-\lambda)\, x_2(\lambda)\, d\lambda$$

$$\Big\} \quad ay_1(t) + by_2(t) = \int_0^t (t-\lambda) x_1(\lambda)\, d\lambda + b\int_0^t (t-\lambda) x_2(\lambda)\, d\lambda$$

$$ax_1(t) + bx_2(t) \Rightarrow y_3(t) = \int_0^t (t-\lambda)\left(ax_1(\lambda) + bx_2(\lambda)\right) d\lambda$$

Since $y_3(t) = ay_1(t) + by_2(t)$, the system is <u>linear</u>

(f) following same procedure of part (e), the system is <u>Linear</u>

(6) <u>1.20</u>

(a) $x(t-t_1) \rightarrow |x(t-t_1)|$

$y(t-t_1) = |x(t-t_1)| \Rightarrow$ the system is <u>time-invariant</u>

(b) $x(t-t_1) \rightarrow e^{x(t-t_1)}$
$y(t-t_1) = e^{x(t-t_1)}$
} time-invariant

(c) $x(t-t_1) \rightarrow (\sin t)\, x(t-t_1)$
$y(t-t_1) = (\sin(t-t_1))\, x(t-t_1)$
} time-varying

(d) $x(t-t_1) \rightarrow \begin{cases} x(t-t_1) & \text{when } |x(t-t_1)| \le 10 \\ 10 & \text{when } |x(t-t_1)| > 10 \end{cases}$

$y(t-t_1) = \begin{cases} x(t-t_1) & \text{when } |x(t-t_1)| \le 10 \\ 10 & \text{when } |x(t-t_1)| > 10 \end{cases}$
} time-invariant

(e) $x(t-t_1) \rightarrow \int_0^t (t-\lambda)\, x(\lambda-t_1)\, d\lambda$

$y(t-t_1) \rightarrow \int_0^{t-t_1} (t-t_1-\lambda)\, x(\lambda)\, d\lambda$

Let $\tilde{\lambda} = t_1+\lambda \rightarrow \lambda = \tilde{\lambda} - t_1$

$\therefore y(t-t_1) = \int_{t_1}^{t} (t-\tilde{\lambda})\, x(\tilde{\lambda}-t_1)\, d\tilde{\lambda}$
} time-varying

(f) Follow same procedure of part (e): the system is time-varying

(7) 1.29

(a) Casual and has memory

Linearity?

$x_1[n] \rightarrow y_1[n] = x_1[n] + 2x_1[n-2]$

$x_2[n] \rightarrow y_2[n] = x_2[n] + 2x_2[n-2]$

$ax_1[n] + bx_2[n] \rightarrow y_3[n] = ax_1[n] + bx_2[n] + 2(ax_1[n] + bx_2[n])$

$ay_1[n] + by_2[n] = y_3[n] \Rightarrow$ Linear

Time-Invariant?

$x[n-n_1] \rightarrow x[n-n_1] + 2x[n-n_1-2]$

$y[n-n_1] = x[n-n_1] + 2x[n-n_1-2]$

$\Rightarrow$ time-invariant

(b) The system non-causal and has a memory since it depends on future input.

The system is linear and time-invariant (follow same procedure of part (a))

(c) Causal and memoryless since the output depends only on the input at time n.

Linear? $x_1[n] \rightarrow y_1[n] = n x_1[n]$

$x_2[n] \rightarrow y_2[n] = n x_2[n]$

$ax_1 + bx_2 \rightarrow y_3[n] = n\left(ax_1[n] + bx_2[n]\right)$

$ay_1[n] + by_2[n] = y_3[n] \Rightarrow$ Linear

Time-Invariant?

$x[n-n_1] \rightarrow n\,x[n-n_1]$

$y[n-n_1] = (n-n_1)\,x[n-n_1]$

$\Rightarrow$ time-varying

(d) The system is caused, memoryless, linear, and time varying

Follow same procedure of part (c)

(e) The system is caused, memoryless, nonlinear, and time-invariant

(f) The system is caused, memoryless, nonlinear, and time-invariant

(g) The system is caused, has memory, and linear

Time-invariant? It is time-varying (see 1.20(f))